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# Confinement model for FRP confined normal- and high-strength concrete circular columns

## Abstract

This study establishes a confinement model for FRP confined normal- and high-strength concrete circular columns. A new column parameter was suggested for estimating the compressive strength of FRP confined concrete. The proposed model is able to predict the ultimate condition of FRP confined concrete columns that have similar unconfined concrete strength and confining pressure but significant differences in the jacket stiffness. The proposed model was then verified using a database of 574 FRP confined concrete circular columns with different types of FRP. This database covers unconfined concrete strength between 15 MPa and 170 MPa and specimens with a diameter ranging from 51 mm to 406 mm. Furthermore, this database includes specimens with a variety of FRP types: carbon FRP (CFRP), glass FRP, high-modulus carbon FRP, aramid FRP (AFRP), CFRP tube, ultra-high-modulus CFRP tube, and AFRP tube. Finally, the model's prediction fits the experimental results very well by verifying the proposed model with the extensive database.

## Keywords

columns, confinement, strength, model, frp, concrete, confined, circular, normal, high

## Disciplines

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# **Confinement Model for FRP Confined Normal- and High-Strength Concrete Circular Columns**

Thong M. Pham<sup>1</sup> and Muhammad N.S. Hadi<sup>2</sup>

## **Abstract**

This study establishes a confinement model for FRP confined normal- and high-strength concrete circular columns. A new column parameter was suggested for estimating the compressive strength of FRP confined concrete. The proposed model is able to predict the ultimate condition of FRP confined concrete columns that have similar unconfined concrete strength and confining pressure but significant differences in the jacket stiffness. The proposed model was then verified using a database of 574 FRP confined concrete circular columns with different types of FRP. This database covers unconfined concrete strength between 15 MPa and 170 MPa and specimens with a diameter ranging from 51 mm to 406 mm. Furthermore, this database includes specimens with a variety of FRP types: carbon FRP (CFRP), glass FRP, high-modulus carbon FRP, aramid FRP (AFRP), CFRP tube, ultra-high-modulus CFRP tube, and AFRP tube. Finally, the model's prediction fits the experimental results very well by verifying the proposed model with the extensive database.

**Keywords:** Fiber reinforced polymer; Confinement; Concrete columns; Compressive strength; Strain; High-strength concrete.

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## **1. Introduction**

Fiber-reinforced polymer (FRP) has been commonly used in practice as a confining material for concrete columns to enhance significantly their strength and ductility. The use of FRP has been applied to both normal strength concrete (NSC) and high strength concrete (HSC) under both concentric and eccentric loads [1-6]. This use of FRP in industry has required design guidelines for these applications. Many confinement models for FRP confined concrete columns, therefore, were proposed to simulate the behavior of confined concrete columns [7-14]. Most of the existing models is applicable for FRP confined NSC columns with exceptions of models by Mandal et al. [15], Cui and Sheikh [16], Berthet et al. [7], and Xiao et al. [17]. It is noted that the model by Xiao et al. [17] was developed from actively confined HSC. Therefore, it is necessary to develop a model covering a wide range of the unconfined concrete strength from NSC to HSC.

This study first conducts an overview about the existing FRP confined concrete models for circular sections. Column parameters affecting the ultimate condition of FRP confined concrete are discussed. A new column parameter then is suggested to take into account for estimating the compressive strength of FRP confined concrete. The proposed strength model was established based on principles of artificial neural networks (ANN) while the proposed strain model was developed by using the energy approach. This study also collated an extensive database with varied FRP types to calibrate and verify the proposed model.

## **2. Mechanism of confinement**

In this section, key assumptions of existing strength models are studied and discussed. The most popular form for calculating the compressive strength of confined concrete, which was

proposed by most of the existing strength models for FRP confined concrete, based on the following form:

$$\frac{f'_{cc}}{f'_{co}} = 1 + k_1 \frac{f_l}{f'_{co}} \quad (1)$$

where  $f'_{cc}$  and  $f'_{co}$  are the compressive strength of confined concrete and unconfined concrete, respectively,  $f_l$  is the lateral confining pressure, and  $k_1$  is the confinement effectiveness coefficient. The above form proposed by Richart et al. [18] based on tests of actively confined concrete with a value of 4.1 for  $k_1$ . The confining pressure could be calculated as follows:

$$f_l = \frac{2f_f t}{d} \quad (2)$$

where  $f_f$  is the tensile strength of FRP determined from flat coupon tests,  $t$  is the thickness of FRP, and  $d$  is the section diameter.

In addition, a few of the existing models had proposed a modified format of the above equation in the following form:

$$\frac{f'_{cc}}{f'_{co}} = 1 + k_1 \left( \frac{f_l}{f'_{co}} \right)^n \quad (3)$$

where  $n$  is calibrated from a database.

In general, the value of  $k_1$  could be a constant or a function of confinement ratio ( $f_l/f'_{co}$ ). This coefficient was attained by calibration of a database [7, 10-12, 19-24]. So, the accuracy of this coefficient depends on the size and reliability of the database used in their calibration. Ozbakkaloglu et al. [25] have conducted an overview of 88 confinement models for FRP confined concrete in circular sections. From that study, it can be seen that majority of equations for calculating the compressive strength of confined concrete was a function of the unconfined concrete strength ( $f'_{co}$ ) and the confining pressure ( $f_l$ ).

Exceptionally, some other models from the literature presented their models coming up with different forms or approaches. Mander et al. [26] proposed a different form developed from a multiaxial failure surface for estimating the compressive strength of confined concrete as follows:

$$f'_{cc} = f'_{co} \left( -1.254 + 2.254 \sqrt{1 + \frac{7.94 f_l}{f'_{co}}} - 2 \frac{f_l}{f'_{co}} \right) \quad (4)$$

Mohr-Columb failure criterion was adopted by Shehata et al. [27] and Li et al. [28] to propose the following equation:

$$f'_{cc} = f'_{co} + f_l \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \quad (5)$$

where  $\phi$  is the angle of internal friction of concrete.

Wu and Zhou [13] adopted Hoek-Brown failure criterion[29] to propose the following equation for strength estimation:

$$\frac{f'_{cc}}{f'_{co}} = \frac{f_l}{f'_{co}} + \sqrt{\left( \frac{16.7}{(f'_{co})^{0.42}} - \frac{(f'_{co})^{0.42}}{16.7} \right) \frac{f_l}{f'_{co}} + 1} \quad (6)$$

As aforementioned, there are a few approaches to develop an equation for strength estimation of confined concrete. All of the above models assumed that the compressive strength of confined concrete is a function of the unconfined concrete and the confining pressure. In the following sections, this study indicates some specimens, which have been reported in the database and conducted by different researchers, had very similar values of unconfined concrete strength and confining pressure but they had significant difference in the compressive strength of confined concrete.

### 3. Test database

#### 3.1. General

The test database used in this study contains experimental results of 574 FRP confined circular plain concrete columns. The test database is collated from several experimental tests being conducted over the past few decades [5, 8-10, 15, 17, 21, 23, 27, 28, 30-60]. Other tests were conducted by Evans et al. (2008), Howie and Karbhari (1994), Ilki et al. (2002), Issa and Karam (2004), Miyauchi et al. (1999), Ongpeng (2006), Owen (1998), Rousakis et al. (2003), Suter and Pinzelli (2001), Comert et al. (2009), Micelli et al. (2001), Rousakis (2001), Chan et al. (2007), Kashyap (2007), and Wang (2008) as cited in Ozbakkaloglu and Lim [61]. The database contains a variety of FRP types: carbon FRP (CFRP, 317 specimens), glass FRP (GFRP, 119 specimens), high-modulus carbon FRP (HM CFRP, 45 specimens), aramid FRP (AFRP, 35 specimens), CFRP tube (28 specimens), ultra-high-modulus CFRP (UHM CFRP, 7 specimens) tube, and AFRP tube (23 specimens). The unconfined concrete strength in the database ranges between 15 MPa and 170 MPa. The diameter of specimens varies between 51 mm and 406 mm. The test database covers a wide range of FRP confinement levels ( $f_l/f_{co}'$ ). The compressive strength of the most heavily confined specimen and the least lightly confined specimen increased by about 440% and 10%, respectively. All specimens exhibiting a stress-strain curve with a descending branch, which is defined in a subsequent section, and a negligible strength increase were excluded from the database. Specimens damaged by premature rupture of FRP were also excluded from the database.

#### 3.2. Stress-strain curve

The stress-strain relationship of FRP confined concrete is classified into two types including an ascending branch and a descending branch as shown in Fig. 1. This study only concerns

the ascending branch type specimens. There are three key points of the stress-strain curve which are the ultimate strength of unconfined concrete, the transition point, and the ultimate strength of the confined concrete. The stress-strain relationship proposed by Lam and Teng [11] is widely accepted and was adopted by ACI440.2R [62]. So this stress-strain relationship was nominated herein for FRP confined circular plain concrete specimens. It is also commonly accepted that the stress-strain relation is bi-linear. If so, once the ultimate stress/strain is precisely determined, the stress-strain curves should not be much different even though they could be calculated by different models. Therefore, determining the ultimate condition of FRP confined concrete plays an important role in a confinement model.

### ***3.3. Effect of confinement stiffness***

As mentioned above, many of the existing models have confirmed that only the unconfined concrete strength and the confining pressure affect significantly the compressive strength of FRP confined concrete. However, Teng et al. [63] indicated that there were specimens with very similar unconfined concrete strength and confining pressure but they yielded different compressive strength of confined concrete. Teng et al. [63] then introduced two new parameters that affect the compressive strength of confined concrete. The two new parameters are the confinement stiffness ratio ( $\rho_k$ ) and the strain ratio ( $\rho_\varepsilon$ ) as estimated follows:

$$\rho_k = \frac{2E_f t}{\frac{f'_{co}}{\varepsilon_{co}} d} \quad (7)$$

$$\rho_\varepsilon = \frac{\varepsilon_{fe}}{\varepsilon_{co}} \quad (8)$$



where  $E_f$  is the elastic modulus of FRP,  $\varepsilon_{co}$  is the axial strain at peak stress of unconfined concrete, and  $\varepsilon_{fe}$  is the actual rupture strain of FRP in the hoop direction. The compressive strength of confined concrete was calculated as follows:

$$\frac{f'_{cc}}{f'_{co}} = 1 + 3.5(\rho_k - 0.01)\rho_\varepsilon \quad (9)$$

The equation above is used for specimens having  $\rho_k$  greater than 0.01. **Table 1** was extracted from the full database to show that even though some specimens had very similar unconfined concrete strength and confining pressure, these specimens had significant difference in the compressive strength of confined concrete. Due to space limitation, only six groups of typical test results are presented in the table. It is noted that the specimens in each group of **Table 1** were conducted by different researchers and are named “pseudo-identical specimens”. These specimens had different confinement stiffness ratio and strain ratio. As a result, their strengths were different as based on Teng et al.’s model [63]. It means that two new parameters proposed by Teng et al. [63] seem reasonable.

Interestingly, **Eq. 9** shows that the compressive strength of confined concrete must be in direct proportion to the value of  $(\rho_k - 0.01)\rho_\varepsilon$ . It is found in Group 2 of **Table 1** that the Columns 2a had the value of  $(\rho_k - 0.01)\rho_\varepsilon$  being greater than that of Columns 2b but the value of  $f'_{cc}$  of Columns 2a is much smaller than that of Columns 2b. Similarly, the columns in Groups 3 and 6 also have the same indication. It indicates that using the two new parameters may not well predict the compressive strength of confined concrete. In such cases, the FRP thickness ratio ( $t/d$  in **Table 1**) can reflect the difference in these “pseudo-identical specimens” that have the same unconfined concrete strength and confining pressure but different compressive strength of confined concrete. The FRP thickness ratio always is in direct proportion to the compressive strength of confined concrete as confirmed by the database. Furthermore, Pham

and Hadi [64] also took the FRP thickness ratio ( $t/d$ ) into account for calculating the compressive strength of FRP confined rectangular columns. Therefore, this study takes into account the unconfined concrete strength, the confining pressure and the FRP thickness ratio to calculate the compressive strength of confined concrete.

#### 4. Strength model for FRP confined circular concrete columns

##### 4.1. The proposed strength model

In order to calculate the compressive strength of FRP confined concrete columns, this study adopted a method proposed by Pham and Hadi [65] to generate a simple-form equation from a trained ANN. Pham and Hadi [65] concluded that the output could be calculated by multiplying the inputs by a proportional matrix if the proposed ANN is trained and yields good results. The inputs herein are column parameters of FRP confined concrete while the outputs are its compressive strength. It means that the compressive strength of confined concrete could be predicted by multiplying the column parameters by the proportional matrix. The proportional matrix is resulted from multiplying the input weight matrix by the layer weight matrix and normalization process of the inputs and the outputs. Details of this method and definitions of some concepts could be found in the studies by Pham and Hadi [65].

As mentioned above, the ratio between the FRP thickness and the section diameter affects the compressive strength of FRP confined concrete. Thus, three column parameters are considered in this model, which are the unconfined concrete strength  $f_{co}'$  (MPa), the confining pressure  $f_l$  (MPa), and the ratio between the FRP thickness and the section diameter  $t/d$  (%). The compressive strength of FRP confined circular columns could be calculated as follows:

$$f_{cc}' = 0.7f_{co}' + 1.8f_l + 5.7\frac{t}{d} + 13 \quad (10)$$

where the confining pressure  $f_l$  is estimated from [Eq. 2](#).

The proposed model could be used to estimate the compressive strength of confined concrete made of a variety of FRP types including CFRP, GFRP, AFRP, HM CFRP, UHM CFRP tube, CFRP tube, and AFRP tube. In addition, it is noted that Eq. 10 covers columns that have parameters in the ranges shown in Table 2.

#### 4.2. Verification of the strength model

The proposed strength model was verified by the database including seven types of FRP and a wide range of compressive strength of unconfined concrete. Fig. 2 shows 574 data points of the predicted compressive strength of FRP confined concrete versus their experimental results. As can be seen in Fig. 2, the proposed strength model could predict the compressive strength of FRP-confined normal- and high-strength concrete circular columns. The compressive strength of unconfined concrete ranges between 15 MPa and 170 MPa. In addition, the normalized predicted compressive strengths of confined columns versus their normalized experimental results are shown in Fig. 3. Five existing models were studied in this verification [7, 12, 14, 63, 66]. The average absolute error (AAE), which is calculated using Eq. 11, of the above models ranges between 11.54% and 15.97%. The proposed model shows the smallest value of AAE (9.88%) among these models.

$$AAE = \frac{\sum_{i=1}^N \left| \frac{pre_i - exp_i}{exp_i} \right|}{N} \quad (11)$$

where  $pre$  is the model predictions,  $exp$  is the experimental results, and  $N$  is the total number of specimens. The mean square error (MSE) and the standard deviation (SD) are also calculated to assess the accuracy of the models:

$$MSE = \frac{\sum_{i=1}^N \left( \frac{pre_i - exp_i}{exp_i} \right)^2}{N} \quad (12)$$

$$SD = \sqrt{\frac{\sum_{i=1}^N \left( \frac{pre_i}{exp_i} - \frac{pre_{avg}}{exp_{avg}} \right)^2}{N-1}} \quad (13)$$

Furthermore, the equation of the best-fit line (in a format of  $y=ax$ ) and the correlation factor ( $R^2$ ) in each model are reported. The five models above have a correlation factor ranging from 0.76 to 0.79 while that factor of the proposed models is 0.88. The value of “ $a$ ” of the equation of the best-fit line could depict that the model is conservative ( $a < 1$ ) or vice versa ( $a > 1$ ). As can be easily seen in Fig. 3 that the models by Berthet et al. [7], Teng et al. [63], Wu and Wang [66], and Yazici and Hadi [14] are conservative while the model by Matthys et al. [12] is less-conservative. Interestingly, the value of “ $a$ ” of the proposed strength model is equal to 1. In addition, the error of the strength models was statistically verified and presented in Fig. 4.

#### 4.3. Discussion the proposed strength model

Figs. 3-4 show that Eq. 10 compares well with the experimental data. However, the constant (value of 13) at the end of the equation may cause a considerable error in estimating the compressive strength of a specimen having low unconfined concrete strength and confining pressure. As a sequence, the following equation is proposed as an approximation:

$$f'_{cc} = 0.91f'_{co} + 1.88f_l + 7.6 \frac{t}{d} \quad (14)$$

To verify the performance of Eq. 14 versus the experimental data, Fig. 5 shows the prediction results versus the experimental results. The error calculated from the prediction of Eq. 14 slightly increase as compared to that of Eq. 10. To extend the applicability of the proposed models, larger sizes of specimens which may be available in the future could be used to retrain and test the models. When the proposed model has been properly trained, verified, and

tested with a comprehensive experimental database, it can be used with a high degree of confidence.

## 5. Strain model for FRP confined circular concrete columns

### 5.1. Strain energy absorption

The proposed strain model was developed based on the energy approach that was proposed by Pham and Hadi [67]. Pham and Hadi [67] concluded that there is a linear relationship between the additional volumetric strain energy absorbed by a column core ( $U_{cc}$ ) and the volumetric strain energy absorbed by FRP ( $U_f$ ). When the strain of confined concrete is below the peak strain of the corresponding unconfined concrete, the effect of FRP is negligible. Thus, it is assumed that the additional energy in the column core equals the area under the experimental stress–strain curves starting from the value of unconfined concrete strain. The additional volumetric strain energy absorbed by a column core ( $U_{cc}$ ) is calculated as follows:

$$U_{cc} = \int_{\varepsilon_{co}}^{\varepsilon_{cc}} f_c d\varepsilon_c = \frac{(\varepsilon_{cc} - \varepsilon_{co})(f'_{co} + f'_{cc})}{2} \quad (15)$$

where  $f_c$  is the axial stress of the concrete,  $d\varepsilon_c$  is an increment of the axial strain,  $U_{cc}$  is the volumetric strain energy of confined concrete,  $\varepsilon_{cc}$  is the ultimate strain of confined concrete. The volumetric strain energy absorbed by FRP ( $U_f$ ) could be determined as follows:

$$U_f = \rho_f \left( \frac{1}{2} f_{fe} \varepsilon_{fe} \right) \quad (16)$$

Where  $\rho_f$  is the volumetric ratio of FRP and calculated as shown in Eq. 17,  $U_f$  is the volumetric strain energy of FRP, and  $f_{fe}$  and  $\varepsilon_{fe}$  are the actual rupture strength and rupture strain of FRP on the columns, respectively.

$$\rho_f = \frac{4t}{d} \quad (17)$$

### 5.2. The proposed strain model

The proposed strain model requires specimens including the actual rupture strain of FRP reported. Test results of specimens not reporting the actual rupture strain of FRP were excluded from the database. Thus, a new database (215 specimens) extracted from the full database was used to develop the proposed strain model. The new database covers unconfined concrete strength between 24 MPa and 112 MPa, and a variety of FRP types including CFRP, GFRP, AFRP, HM CFRP, CFRP tube, and AFRP tube. The energy absorption of 215 specimens was determined using Eqs. 15 and 16, and the results are presented in Fig. 6. Next, a regression analysis was undertaken to attain the following equation:

$$U_{cc} = 6.62 U_f \quad (18)$$

Substituting Eqs. 15 and 16 into Eq. 18, results in the following equation:

$$\varepsilon_{cc} = \varepsilon_{co} + \frac{4 k t f_{fe} \varepsilon_{fe}}{d (f'_{co} + f'_{cc})} \quad (19)$$

where the proportion coefficient  $k$  is equal to 6.62. This expression could be used to calculate the axial strain of CFRP confined concrete columns in circular sections. Using this calculated strain, any model could be utilized to calculate the confined concrete strength. Lam and Teng model [11] was adopted to express another form of Eq. 19 as follows:

$$\varepsilon_{cc} = \varepsilon_{co} + \frac{2 k t f_{fe} \varepsilon_{fe}}{d f'_{co} + 3.3 f_{fe} t} \quad (20)$$

### 5.3. Verification of the strain model

Fig. 7 shows theoretical strain versus experimental strain of FRP confined circular columns. This figure depicts that the proposed strain model predicts both the 132 NSC columns and the 83 HSC columns. This figure also shows that the proposed model's prediction of columns with low compressive strain is closer to experimental results than the others. The errors of the model's prediction in a range of high strain could be explained through Fig. 6, which describes

that the relationship between the two energies of columns having low energy absorption is more correlative than columns having high energy absorbed.

In addition, a total of 215 data points are plotted in Fig. 8 to assess the performance of existing models and the proposed model. Five strain models were considered in this verification [11, 14, 63, 68, 69]. Performance and accuracy of all six models are comparable with the value of AAE ranging from 19% to 36%. Among the above models, the proposed strain model shows the best performance with the AAE of 19%. In general trend, the model by Teng et al. [63] and the proposed model depict good prediction for all ranges of columns' strain. The model by De Lorenzis and Tepfers [68] also shows very good agreement between prediction and experimental results. However, this model is very conservative in a range of columns having high compressive axial strain. In addition, the error of the strain models was statistically verified and presented in Fig. 9.

## **6. Conclusions**

A confinement model was developed for FRP confined normal- and high-strength concrete columns. The predictions of the proposed model fit the experimental results from the extensive database very well. The findings presented in this paper are summarized as follows:

1. In order to calculate the compressive strength of FRP confined circular columns, the ratio between the thickness of FRP and the column diameter should be taken into account. When this parameter is considered, only an unified equation is used to calculate the compressive strength of confined concrete with varied FRP types, which have significant difference in the jacket stiffness.
2. The proposed model could estimate the compressive strength of confined concrete with unconfined concrete strength ranging between 15 MPa and 170 MPa.

3. This study confirms that using energy method could estimate well the compressive strain of FRP confined concrete as compared to experimental results. This study refines the applicability of the energy-based strain model proposed by Pham and Hadi [67] from only CFRP to seven types of FRP.

Finally, this study takes the FRP thickness ratio into account to predict the compressive strength of FRP confined circular concrete columns. This consideration provides an unified equation covering all types of FRP and diminishes the disadvantage of most of existing models that proposed each experimental factor for each FRP type.

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### **Notations**

- $AAE$  = average absolute error;  
 $d$  = diameter of circular sections;  
 $d\varepsilon_c$  = an increment of the axial strain;  
 $E_f$  = elastic modulus of FRP;  
 $f_c$  = axial stress of concrete;  
 $f_f$  = tensile strength of FRP;  
 $f_{fe}$  = actual tensile stress of FRP;  
 $f_l$  = confining pressure of FRP confined concrete columns;  
 $f_{cc}'$  = confined concrete strength;  
 $f_{co}'$  = unconfined concrete strength;  
 $k$  = proportion coefficient;  
 $k_l$  = confinement effectiveness coefficient;  
 $MSE$  = mean square error;  
 $n$  = experimental coefficient factor;



$N$  = total number of the test data;  
 $SD$  = standard deviation;  
 $t$  = thickness of FRP;  
 $U_{cc}$  = additional volumetric strain energy of confined concrete;  
 $U_f$  = volumetric strain energy of FRP;  
 $\varepsilon_{fe}$  = actual strain of FRP at rupture;  
 $\varepsilon_{cc}$  = ultimate axial strain of confined concrete;  
 $\varepsilon_{co}$  = axial strain of the unconfined concrete at the maximum stress;  
 $\rho_f$  = volumetric ratio of FRP;  
 $\rho_k$  = confinement stiffness ratio;  
 $\rho_\varepsilon$  = strain ratio; and  
 $\phi$  = angle of internal friction of concrete.

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Table 1. Pseudo-identical specimens in the database

No. of group	Sources	$d$ (mm)	$f'_{co}$ (MPa)	$f_l$ (MPa)	$t/d$ (%)	$f'_{cc}$ (MPa)	$\rho_k$	$\rho_\varepsilon$	$(\rho_k - 0.01)\rho_\varepsilon$
1a	Mirmiran et al. 1998	152	29.8	19.5	0.5	63.0	0.04	9.4	0.27
	Mirmiran et al. 1998	152	29.8	19.5	0.5	58.7	0.04	9.4	0.27
	Mirmiran et al. 1998	152	31.2	19.5	0.5	63.1	0.04	9.2	0.25
	Mirmiran et al. 1998	152	31.2	19.5	0.5	65.4	0.04	9.2	0.25
	Silva and Rodrigues 2006	150	31.1	19.9	1.7	91.6	0.07	8.3	0.49
1b	Silva and Rodrigues 2006	150	29.6	19.9	1.7	89.4	0.07	5.0	0.32
	Silva and Rodrigues 2006	150	31.1	19.9	1.7	87.5	0.07	7.9	0.47
	Silva and Rodrigues 2006	150	31.1	19.9	1.7	91.9	0.07	7.8	0.46
	Silva and Rodrigues 2006	150	29.6	19.9	1.7	89.8	0.07	5.0	0.32
	Silva and Rodrigues 2006	150	31.2	19.9	1.7	91.9	0.07	8.0	0.48
2a	Xiao and Wu 2000	152	33.7	23.7	0.8	82.9	0.09	4.1	0.34
	Xiao and Wu 2000	152	33.7	23.7	0.8	86.2	0.09	4.5	0.38
2b	Lin and Chen 2001	120	32.7	22.3	1.5	101.3	0.06	6.3	0.31
	Lin and Chen 2001	120	32.7	22.3	1.5	104.5	0.06	6.3	0.31
3a	Jiang and Teng 2007	152	45.9	12.3	0.3	64.6	0.03	6.6	0.12
	Jiang and Teng 2007	152	45.9	12.3	0.3	65.9	0.03	8.0	0.15
	Youssef et al. 2007	152	44.1	12.5	1.5	80.4	0.03	5.3	0.10
3b	Youssef et al. 2007	152	44.1	12.5	1.5	80.0	0.03	5.3	0.10
	Youssef et al. 2007	152	44.1	12.5	1.5	81.1	0.03	5.3	0.10
4a	Cui and Sheikh 2010	152	48.1	21.5	1.3	109.4	0.10	4.4	0.41
	Cui and Sheikh 2010	152	48.1	21.5	1.3	126.7	0.10	5.5	0.51
4b	Tamuzs et al. 2008	150	48.8	20.4	0.2	72.1	0.05	1.8	0.08
	Tamuzs et al. 2008	150	48.8	20.4	0.2	72.6	0.05	1.5	0.07
5a	Cui and Sheikh 2010	152	48.1	32.2	2.0	162.7	0.15	5.2	0.76
	Cui and Sheikh 2010	152	48.1	32.2	2.0	153.6	0.15	4.7	0.68
5b	Rousakis et al. 2003	150	49.2	30.6	0.3	100.6	0.05	6.2	0.28
6a	Aire et al. 2010	150	69	36.5	0.5	156.0	0.08	4.3	0.29
6b	Xiao et al. 2010	152	70.8	36.7	0.7	180.5	0.14	2.0	0.27

Table 2. Statistics of the column parameters for the proposed strength model

	Max	Min
$f_{co}'$ (MPa)	170	15
$f_l$ (MPa)	109	3
$t/d$ (%)	3.9	0.06
$f_{cc}'$ (MPa)	296	37



















